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Open Supermembranes Coupled to M-Theory Five-Branes

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Abstract

We consider open supermembranes in eleven dimensions in the presence of closed M-Theory five-branes. It has been shown that, in a flat space-time, the world-volume action is kappa invariant and preserves a fraction of the eleven dimensional supersymmetries if the boundaries of the membranes lie on the five-branes. We calculate the reparametrisation anomalies due to the chiral fermions on the boundaries of the membrane and examine their cancellation mechanism. We show that these anomalies cancel with the aid of a classical term in the world-volume action, provided that the tensions of the five-brane and the membrane are related to the eleven dimensional gravitational constant in a way already noticed in M-Theory.

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1 Introduction

Since the recent advent of M-Theory as a step towards a unified description of the five previously known superstring theories, the role of extended objects generalising strings has been reappraised [1]. The p-branes, whose origin stems from the corresponding (p+1)-forms and their duals in the low energy spectrum of the various string theories, play a fundamental role in many non-perturbative phenomena and are beleived to stand on the same footing as the original string. The extended objects in the eleven dimensional M-Theory consist of a membrane coupled to a three-form in the spectrum of the eleven dimensional supergravity and a five-brane coupled to a six-form dual to the three-form. The dynamics of these six dimensional objects have recently received much attention [2, 3, 4, 5, 6, 7, 8, 9, 10], the non-linear Lagrangian whose double dimensional reduction leads to the type II-A D-four-brane action have been written in a covariant [7, 8] and non-manifestly covariant manner [9, 10]. A particular emphasis has been shed on the role of reparametrisation anomalies and their cancellation [11, 12, 14].

The existence of closed supermembranes in eleven dimensions has been known for a few years [15, 16] leading by double dimensional reduction to the type II-A string in ten dimensions [17]. A similar construction for open supermembranes whose boundaries live on a ten dimensional hypersurface was considered in [18]. It was shown that the only anomaly free configuration was the Horava-Witten one where the eleventh dimension is compactified on an orbifold S^1/Z_2 with an $E_8 \times E_8$ gauge group living on the boundaries [19, 20].

In the present paper we further examine the configuration where the eleven dimensional supermembrane couples to five-branes via the strings living on its boundaries. This configuration has received much attention recently especially in relation with tensionless non-critical strings in six dimensions [21, 22, 23, 24, 25, 26, 27]. The goal of this paper is to calculate the reparametrisation anomaly localised at the boundaries of the membrane and examine their cancellation mechanism. In sections II and III we describe the dynamics of the supermembrane in the presence of five-branes. We shall use the non-covariant formulation of the five-brane action and show the influence of the boundaries. A particular emphasis is put on the modification of the various Bianchi identities; the five-branes play the role magnetic charges for the M-Theory three-form and the strings on the boundaries of the membranes are electric and magnetic sources for the five-brane self-dual two-form. In section IV we show that the Bianchi identities imply that the classical action is not invariant under reparametrisation. In section V we calculate the reparametrisation anomalies due to the chiral fermions on the strings. We show that the anomalies are given by the Euler class of two SO(4) bundles: the normal bundle of the string in the five-brane and the normal bundle in eleven dimensions of a seven dimensional manifold containing both the membrane and the five-brane. We show how these anomalies are cancelled by the classical term in the world-volume action describing the coupling of the membrane and the

five-brane without introducing new fields nor modifying the Bianchi identities. The anomaly cancellation mechanism allows to rederive the relations between the membrane tension, the five-brane tension and the eleven-dimensional gravitational constant. These relations have been previously obtained in different contexts [11, 28, 29, 18].

2 The Coupling Between a Supermembrane and a Five-Brane

In reference [18] it was shown that the kappa-invariant action of the open supermembrane in eleven dimensions is given by

$$S = -T_3 \left[\int_{\Sigma_3} \sqrt{-g} + \int_{\Sigma_3} C - \int_{\partial\Sigma_3} B \right], \tag{1}$$

where g_{ij} is the induced metric on the world-volume, C is the pullback of the eleven dimensional super three-form. We consider a supermembrane which has the topology of $\Sigma_2 \times I$ where I is the unit interval and Σ_2 a closed string. The boundaries of the membrane correspond to two closed strings

$$\partial\Sigma_3 = \Sigma_2^1 \cup \Sigma_2^2. \tag{2}$$

For the action (1) to preserve a fraction of the supersymmetries while being kappa-invariant, the boundaries have to lie of an even-dimensional submanifold where lives a two-form [18]. The two-form is essential to preserve the gauge invariance under

$$C \to C + d\Lambda, \tag{3}$$

with Λ an arbitrary two-form. The action (1) is invariant under (3) if the two-form B transforms as

$$B|_{\partial\Sigma_3} \to B|_{\partial\Sigma_3} - \Lambda|_{\partial\Sigma_3}. \tag{4}$$

The five-brane, being six dimensional, kappa-invariant, and having a two-form with the correct transformation rule under the gauge transformation (3) is a natural candidate where the boundaries of the membrane can lie. The interpretation of the eleven-dimensional five-brane as a Dirichlet Brane for membranes was proposed previously in [2]. The possibility that membranes can end on five-branes was pointed out in [22] using charge conservation arguments.

Since our membrane has two boundaries one has to distinguish between two cases, either the membrane has both of its boundaries on a single five brane (case A) or the two boundaries of the membrane belong to two different parallel five-branes (case B). In case (A) the membrane couples to a single two-form and in case (B) it couples to two different two-forms. We shall find it convenient to

orient the strings with the induced orientation from the five-branes so that in both cases (A) and (B) the boundary term in (1) reads

$$I = \epsilon_1 T_3 \int_{\Sigma_2^1} B + \epsilon_2 T_3 \int_{\Sigma_2^2} B, \tag{5}$$

with $\epsilon_i = 1$ if the induced orientations from the membrane and the five-brane on Σ_2^i coincide and $\epsilon_i = -1$ otherwise.

In the following section we examine the dynamics of the two-form B and derive its Bianchi identity.

3 The Five-Brane self-dual two-form action

We consider a five-brane in eleven dimensions. The bosonic part of the five-brane contains a two-form B with a self-dual field strength (in the linearised approximation). The corresponding supermultiplet is a $(2,0)$ tensor multiplet containing five scalars and two chiral spinors. We concentrate on the part containing the self-dual two-form in the linearised approximation. The construction of the full kappa-invariant supercovariant action of the five-brane has been recently achieved [8]. We use the non-covariant formulation [9] where the Lorentz invariance is not manifest. The world-volume of the five-brane Σ_6 is chosen to be a circle bundle $\pi : \Sigma_6 \to \Sigma_5$ with a base manifold Σ_5 and fibres S^1. Locally the first five coordinates $x^\mu,\ \mu = 0, \ldots, 4$ parametrise the base space and x_5 the fibre. The local form of the self-dual two-form is

$$B = b + a \wedge dx_5 \tag{6}$$

where $b_{\mu\nu}$ is a two-form with indices $\mu, \nu = 0 \ldots 4$ and a_μ is a one-form.

The linearised action of b is given by

$$S_6 = -\frac{T_6}{4} \int_{\Sigma_6} h \wedge (*_6 h - \partial_5 b \wedge dx_5), \tag{7}$$

where $*_6$ is the six dimensional Hodge dual map and

$$h = d_5 b, \tag{8}$$

with d_5 being the exterior derivative with respect to the coordinates of the base manifold Σ_5. The constant T_6 is the five-brane tension. Note that $*_6 h = *_5 h \wedge dx_5$. The equations of motion for b read

$$d_5(*_5 h - \partial_5 b) = 0. \tag{9}$$

From (9) it follows that there exits a one-form a such that

$$*_5\, h = \partial_5 b + d_5 a \tag{10}$$

In particular this implies that

$$H = dB \tag{11}$$

becomes

$$H = h + *_6 h \tag{12}$$

This implies that the action (7) describes a self-dual two-form.

Let us now consider topological defects coupled to the two-form B of the five-brane. The action (7) has to be supplemented with

$$I = T_3 \int_{\Sigma_2} B|_{\Sigma_2}, \tag{13}$$

where Σ_2 is the two-dimensional boundary of a three-manifold Σ_3 (a membrane). The coupling constant T_3 is the membrane tension. The submanifold Σ_2 is supposed to be embedded in Σ_6. One associates to the homology class $[\Sigma_2]$ a cohomology class $\delta(\Sigma_2, \Sigma_6)$ by Poincaré duality such that

$$\int_{\Sigma_2} B|_{\Sigma_2} = \int_{\Sigma_6} B \wedge \delta(\Sigma_2, \Sigma_6). \tag{14}$$

In the following, we find it convenient to choose Σ_5 such that

$$\Sigma_2 \subset \Sigma_5, \tag{15}$$

so I reads

$$T_3 \int_{\Sigma_2} b, \tag{16}$$

and we have

$$\delta(\Sigma_2, \Sigma_6) = \delta(\Sigma_2, \Sigma_5) \wedge dx_5. \tag{17}$$

Using this four form the linearised equations of motion are

$$d_5(*_5 h - \partial b) = \frac{2T_3}{T_6} \delta(\Sigma_2, \Sigma_5). \tag{18}$$

The solution of (18) reads

$$*_5\, h = \partial_5 b + d_5 a + \frac{2T_3}{T_6} \delta^1(\Sigma_2, \Sigma_5). \tag{19}$$

where we have used

$$\delta(\Sigma_2, \Sigma_5) = d_5 \delta^1(\Sigma_2, \Sigma_5), \tag{20}$$

as Σ_2 is a closed surface. These equations of motion describe a self-dual two-form provided one modifies the definition of the field strength

$$H = dB + \frac{2T_3}{T_6} \delta^1(\Sigma_2, \Sigma_6) \tag{21}$$

This implies that

$$H = h + *_6 h, \tag{22}$$

describing a self-dual field strength $H = *_6 H$. We now find that the Bianchi identity becomes

$$dH = \frac{2T_3}{T_6}\delta(\Sigma_2, \Sigma_6) \tag{23}$$

The string world-volume Σ_2 plays the role of an electric and magnetic charge for the five-brane that the Bianchi identity $dH = 0$ is modified. In the presence of a background three-form C the field strenght H is modified as $H \to H - C|_{\Sigma_6}$, so the Bianchi identity (23) becomes

$$dH = \frac{2T_3}{T_6}\delta(\Sigma_2, \Sigma_6) - G|_{\Sigma_6}. \tag{24}$$

The Dirac quantisation condition applied in the six-dimensional five-brane gives

$$\frac{T_3^2}{T_6} = m\pi, \tag{25}$$

with m an arbitrary integer. This relation will be useful in the following.

The presence of the five-brane modifies the Bianchi identity verified by the field strenght G of eleven dimensional supergravity. the Bianchi identity now reads

$$dG = -2\kappa_{11}^2 T_6 \delta(\Sigma_6, Q). \tag{26}$$

Here and in the following Q denotes the eleven-dimensional space-time and $\delta(\Sigma, \Sigma')$ with Σ a d-dimensional submanifold of Σ' is defined by

$$\int_\Sigma \omega = \int_{\Sigma'} \omega \wedge \delta(\Sigma, \Sigma'), \tag{27}$$

for an arbitrary form ω defined in Σ'. For future use we mention the relation

$$d\delta(\Sigma, \Sigma') = (-1)^{d+1}\delta(\partial\Sigma, \Sigma'). \tag{28}$$

Similarly we shall denote by $N(\Sigma, \Sigma')$ the normal bundle of Σ in Σ'. We shall also frequently use a consequence of the Thom isomorphism theorem [30] which reads:

$$\delta(\Sigma, \Sigma')|_\Sigma = \chi\Big(N(\Sigma, \Sigma')\Big), \tag{29}$$

where χ is the Euler class of the normal bundle.

Finally we mention that the Dirac quantisation condition in eleven dimensions gives the relation

$$\kappa_{11}^2 T_6 T_3 = n\pi, \tag{30}$$

with n an arbitrary integer. Note that the two relations (25) and (30) imply that there is only one physical scale in the theory.

4 The classical transformation under reparametrisation

We return to our configuration of a membrane having its two boundaries one one or on two five-branes. We examine separatly the two cases (A) and (B) described in section 2. Let us deal first with the case A where only one five-brane is present. The Bianchi identity for the field strengths H and G read

$$dG = -2\kappa_{11}^2 T_6 \delta(\Sigma_6, Q) \tag{31}$$

and

$$dH = 2\frac{T_3}{T_6}\Big(\epsilon_1\delta(\Sigma_2^1, \Sigma_6) + \epsilon_2\delta(\Sigma_2^2, \Sigma_6)\Big) - G|_{\Sigma_6}. \tag{32}$$

These two equations are compatible as the differential of the right hand side of (32) vanishes. Indeed the strings on the boundary of the membrane are closed so the relation (28) gives $d\delta(\Sigma_2^{1,2}, \Sigma_6) = 0$ and using the Thom isomorphism theorem relating

$$\delta(\Sigma_6, Q)|_{\Sigma_6} = \chi(N(\Sigma_6, Q)) \tag{33}$$

and the fact that the Euler class of a manifold of odd dimension vanishes we get

$$dG|_{\Sigma_6} = 0 \tag{34}$$

With the aid of relation (28) one can integrate the Bianchi identity (31) to get

$$G|_{\Sigma_6} = -2\kappa_{11}^2 T_6 \delta(\Sigma_7, Q) + dC, \tag{35}$$

where Σ_7 is a manifold whose boundary is Σ_6. This leads to the three-form [3]

$$H = dB - C|_{\Sigma_6} + 2\frac{T_3}{T_6}\Big(\epsilon_1\delta^1(\Sigma_2^1, \Sigma_6) + \epsilon_2\delta^1(\Sigma_2^2, \Sigma_6)\Big) + 2\kappa_{11}^2 T_6 \delta^1(\Sigma_7, Q)|_{\Sigma_6}), \tag{36}$$

where

$$\delta(\Sigma_7, Q)|_{\Sigma_6} = d\delta^1(\Sigma_7, Q). \tag{37}$$

Let us now examine the variation of B under a Lorentz gauge transformation. Since G is gauge invariant the gauge variation of C is $C \to C + d\Lambda$. The action is invariant under the transformation (3) so it is possible to choose $\delta C = 0$. As H is gauge invariant we get using relation (29)

$$\delta B|_{\Sigma_2^i} = -2\epsilon_i \frac{T_3}{T_6}\chi_2^1(N_i) - 2\kappa_{11}^2 T_6 \chi_2^1(N')|_{\Sigma_2^i},\ i = 1, 2, \tag{38}$$

[3] If another manifold Σ_7' is chosen, this amounts to modifying $C' = C + \delta(\Sigma_8, Q)$ where the manifold Σ_8 is such that $\partial\Sigma_8 = \Sigma_7 \cup \Sigma_7'$ glued on their common boundary Σ_6 with an opposite orientation, i.e the manifolds Σ_7 and Σ_7' are cobordant.

where $N_i = N(\Sigma_2^i, \Sigma_6)$, $N' = N(\Sigma_7, Q)$, and χ_2^1 is related to χ by the descent equations:

$$\begin{aligned} \chi &= d\chi_3, \\ \delta\chi_3 &= d\chi_2^1. \end{aligned} \tag{39}$$

The classical action is not invariant under the transformations (38). Indeed, the interaction term (5) varies as

$$\begin{aligned} \delta I = &-2\frac{T_3^2}{T_6}\left(\int_{\Sigma_2^1} \chi_2^1(N_1) + \int_{\Sigma_2^2} \chi_2^1(N_2)\right) \\ &-2\kappa_{11} T_6 T_3 \left(\epsilon_1 \int_{\Sigma_2^1} \chi_2^1(N') + \epsilon_2 \int_{\Sigma_2^2} \chi_2^1(N')\right). \end{aligned} \tag{40}$$

The case B can be dealt with in a similar manner. The tensions of the two-five branes, Σ_6^1 and Σ_6^2 are denoted by $T_6^{(i)}, i = 1, 2$. The Bianchi identities of the two three-forms H_1 and H_2 read

$$dH_i = 2\epsilon_i \frac{T_3}{T_6^{(i)}} \delta(\Sigma_2^i, \Sigma_6^i) - G|_{\Sigma_6^i}. \tag{41}$$

The five-branes are magnetic sources for the space-time three-form C, the corresponding Bianchi identity reads:

$$dG = -2\kappa_{11}^2 \left(T_6^{(1)} \delta(\Sigma_6^1, Q) + T_6^{(2)} \delta(\Sigma_6^2, Q)\right). \tag{42}$$

The gauge variation of the two-form B_i then follows

$$\delta B_i|_{\Sigma_2^i} = -2\epsilon_i \frac{T_3}{T_6^{(i)}} \chi_2^1(N_i) - 2\kappa_{11}^2 T_6^{(i)} \chi_2^1(N_i')|_{\Sigma_2^1}. \tag{43}$$

The corresponding variation of the interaction term (5) is then given by an expression which is similar to (40) and reads:

$$\begin{aligned} \delta I = &-2\frac{T_3^2}{T_6^{(1)}} \int_{\Sigma_2^1} \chi_2^1(N_1) - 2\frac{T_3}{T_6^{(2)}} \int_{\Sigma_2^2} \chi_2^1(N_2) \\ &-2\kappa_{11} T_3 T_6^{(1)} \epsilon_1 \int_{\Sigma_2^1} \chi_2^1(N') - 2\kappa_{11} T_3 T_6^{(2)} \epsilon_2 \int_{\Sigma_2^2} \chi_2^1(N'). \end{aligned} \tag{44}$$

In summary, the Bianchi identities imply that the classical world-volumw action is not invariant under reparametrisations. The variation of the action is given in the two cases involving one or two five-branes by equations (40) and (44) respectively.

5 The Reparametrisation Anomaly

The aim of this section is the calculation of the anomaly of the diffeomorphisms that leave the membrane and the five-brane invariant. The closed membrane is anomaly-free so we concentrate on the anomalies that are localised on the intersection between the five-brane Σ_6 and the open membrane Σ_3. The potential source of anomalies are the Green-Schwarz fermions which form a 32-components Majorana spinor of $SO(10,1)$. The reparametrisation anomaly stems from the fermionic zero modes. Let us consider the case of a cylindrical membrane. The zero modes are independent of the transverse direction to the boundary. As such the anomaly receives contributions from the two strings on the boundary of the membrane. This is similar to the anomaly calculation performed in [20]. In the following we calculate the anomaly due to the zero modes in this cylindrical situation. These zero modes are the reduction of the eleven dimensional spinor to the two dimensional surface, Σ_2 equipped with the orientation induced from the five-brane.

Let $N(\Sigma_6, Q)$ be the normal bundle to Σ_6 in Q, then

$$TQ|_{\Sigma_2} = T\Sigma_6|_{\Sigma_2} \oplus N(\Sigma_6, Q)|_{\Sigma_2} \tag{45}$$

where Σ_2 denotes the the two topologically equivalent boundaries of the membrane. A Lorentz transformation that leaves $T\Sigma_6$ invariant is equivalent to an $SO(5,1)$ Lorentz rotation of $T\Sigma_6$ and an $SO(5)$ gauge transformation of $N(\Sigma_6, Q)$. The 32 complex spinors $SO(10,1)$ decompose as $(4_+, 4) + (4_-, 4)$ under $SO(5,1) \times SO(5)$. The kappa-symmetry condition of the five brane eliminates the $(4_-, 4)$ component so the fermions leading to the anomaly are Weyl fermions in the $(4_+, 4)$ representation. The fact that the kappa-symmetry removes the fermions with negative chirality is due to our choice of the orientation of Σ_2 as the one induced from the five-brane.

Similarly let $N(\Sigma_3, Q)$ be the normal bundle to Σ_3 in Q then

$$TQ|_{\Sigma_2} = T\Sigma_3|_{\Sigma_2} \oplus N(\Sigma_3, Q)|_{\Sigma_2} \tag{46}$$

A Lorentz transformation that leaves $T\Sigma_3$ invariant is equivalent to an $SO(2,1)$ Lorentz rotation of $T\Sigma_3$ and an $SO(8)$ gauge transformation of $N(\Sigma_3, Q)$. The 32 of $SO(10,1)$ decomposes as $(2, 8_+) + (2, 8_-)$ under $SO(2,1) \times SO(8)$. The component which is eliminated by the kappa-symmetry depends on the orientation of Σ_2. The kappa-symmetry condition of the membrane eliminates the $(2, 8_-)$ component if the induced orientations from the membrane and the five-brane coincide and we are left with the $(2, 8_+)$ representation, otherwise it is the $(2, 8_+)$ component which is eliminated. We summarise these two cases by stating that it is $(2, 8_\epsilon)$ component which remains.

We are interested in the diffeomorphisms thet leave both $T\Sigma_6$ and $T\Sigma_3$ invariant. They leave $T\Sigma_2 = T\Sigma_6 \cap T\Sigma_3$ invariant. Since $T\Sigma_6 = T\Sigma_2 \oplus N(\Sigma_2, \Sigma_6)$ and

$T\Sigma_3 = T\Sigma_2 \oplus N(\Sigma_2, \Sigma_3)$, the transformations of interest are i) Lorentz rotation of $SO(1,1)$, ii) $SO(4)$ gauge transformations of

$$N(\Sigma_2, Q) \cap T\Sigma_6 \cap N(\Sigma_3, Q) = N(\Sigma_2, \Sigma_6) \equiv N \tag{47}$$

and iii) $SO(4)$ gauge transformations of

$$N(\Sigma_2, Q) \cap N(\Sigma_6, Q) \cap N(\Sigma_3, Q) \equiv N'. \tag{48}$$

This is so because $N(\Sigma_6, Q) \cap T\Sigma_3 = N(\Sigma_6, Q) \cap N(\Sigma_2, \Sigma_3)$ is of real rank one. The bundle N' is restriction to Σ_2 of the normal bundle of a seven manifold Σ_7 such that

$$\Sigma_6 = \partial\Sigma_7, \;\; \Sigma_3 \subset \Sigma_7 \tag{49}$$

and

$$N' = N(\Sigma_7, Q) \cap N(\Sigma_2, Q) \tag{50}$$

We can now determine the representation of the physical Green-Schwarz fermion under the $SO(1,1) \times SO(4) \times SO(4)$ resulting gauge group. These are the representations that are in common from the decomposition of the $(4_+, 4)$ of $SO(5,1) \times SO(5)$ and the $(2, 8_\epsilon)$ of $SO(2,1) \times SO(8)$ under the group $SO(1,1) \times SO(4) \times SO(4)$. Since $(4_+, 4) = (1_+, 2_+, 4) + (1_-, 2_-, 4)$ and $(2, 8_\epsilon) = (2, 2_+, 2_\epsilon) + (2, 2_-, 2_{-\epsilon})$ the fermions are in the representation $(1_+, 2_+, 2_\epsilon) + (1_-, 2_-, 2_{-\epsilon})$ of $SO(1,1) \times SO(4) \times SO(4)$. The left and right handed fermions transform under different representations of the gauge group $SO(4) \times SO(4)$, so there are potential anomalies. The resulting anomaly reads

$$2\pi \left[ch\left(S_+(N) \otimes S_\epsilon(N')\right) - ch\left(S_-(N) \otimes S_{-\epsilon}(N')\right)\right] \hat{A}(T\Sigma_2), \tag{51}$$

where ch is the Chern character, $S_\pm$ is the spin bundle with a given chirality, $\hat{A}$ is the Dirac genus. In order to calculate the Chern character of an $SO(4)$ spin bundle with a given chirality let λ_1 and λ_2 be the Chern roots of the curvature in the fundamental representation of $SO(4)$ then

$$ch(S_\pm) = e^{(\lambda_1 \pm \lambda_2)} + e^{-(\lambda_1 \pm \lambda_2)} = 2 + \frac{\lambda_1^2 + \lambda_2^2 \pm 2\lambda_1\lambda_2}{4} + \ldots. \tag{52}$$

The first Pontryagin class is given by $p_1 = \lambda_1^2 + \lambda_2^2$ and the Euler class is given by $\chi = \lambda_1\lambda_2$ so we get

$$ch(S_\pm) = 2 + \frac{(p_1 \pm 2\chi)}{4}. \tag{53}$$

Finally using the multiplicativity of the Chern character the total anomaly reads

$$I_4 = 4\pi(\chi(N) + \epsilon\chi(N')). \tag{54}$$

We have kept the terms of degree four (the four forms) only. This anomaly has to be equally distributed between the two boundaries of the membrane. We find therefore that the total anomaly reads

$$\delta\Gamma = 2\pi \int_{\Sigma_2^1} (\chi_2^1(N_1) + \epsilon_1 \chi_2^1(N')) + 2\pi \int_{\Sigma_2^2} (\chi_2^1(N_2) + \epsilon_2 \chi_2^1(N')). \tag{55}$$

It is valid in both cases A and B.

This anomaly can be cancelled without introducing new fields. In fact consider the total variation due to the classical term I and to the quantum anomaly, it reads, in the case (A),

$$\begin{aligned}\delta I + \delta\Gamma = &\left(2\pi - 2\frac{T_3^2}{T_6}\right)\left(\int_{\Sigma_2^1} \chi_2^1(N_1) + \int_{\Sigma_2^2} \chi_2^1(N_2)\right) \\ &+ \left(2\pi - 2\kappa_{11}^2 T_3 T_6\right)\left(\epsilon_1 \int_{\Sigma_2^1} \chi_2^1(N') + \epsilon_2 \int_{\Sigma_2^2} \chi_2^1(N')\right).\end{aligned} \tag{56}$$

The variation cancels if

$$\begin{aligned} T_3 T_6 \kappa_{11}^2 &= \pi \\ T_3^2 &= \pi T_6. \end{aligned} \tag{57}$$

This fixes the integers apearing in the Dirac quantisation conditions (25) and (30) to one and is in agreement with other derivations of these relations[11, 28, 29, 18].

Case (B) can be dealt with in a similar manner where we get relations similar to (57) and in addition we get that the two five-brane tensions must be equal.

Notice that the anomaly cancellation mechanism is highly non-trivial due to the structure of $\delta\Gamma$ which is the same as that of δI and to the compatibility requirement with the Dirac conditions.

This mechanism is very different from the one used for the Horava-Witten configuration [19, 18] where extra fermions were required. These fermions are naturally coupled to gauge fields. Here gauge fields are not present in the five-brane spectrum preventing the possible use of extra fermions. With hindsight this reinforces the result that the quantum consistency of the membrane-five-brane coupling follows from the modified Bianchi identities.

We can use the quantum consistency of the membrane coupling to a five-brane in order to derive ten-dimensional configurations which are anomaly free. Suppose that $x^0, \ldots, x^5$ span the five-brane and x^0, x^5, x^6 span the membrane. First of all choose one of the coordinates $x^7 \in S_R^1$ where $R \to 0$, then the M-theory reduces to the type II-A string. The five-brane reduces to a type II-A NS solitonic five-brane and the membrane to a D-2-brane. In the case A the D-2-brane is coupled to a single NS five-brane while in the case B the D-2-brane connects two NS five-branes. A second possibility is to take $x^4 \in S_R^1,\ R \to 0$. The five-brane reduces to a type II-A magnetic D-4-brane and the membrane

to a D-2-brane. A third possibility consists in choosing $x^5 \in S^1_R,\ R \to 0$. The five-brane reduces to a type II-A magnetic 4-brane and the membrane to an open string. Finally, if $x^6 \in S^1_R,\ R \to 0$ along the transverse direction of the membrane, then the five-brane yields a type II-A solitonic NS five-brane and the membrane becomes a tensionless closed string.

6 Conclusion

We have analysed the coupling of the five-brane and the membrane of M-Theory and proved its quantum consistency. We have shown that the Bianchi identities for the four-form of eleven dimensional supergravity and for the self-dual three-form of the five-brane imply that the classical world-volume action is not invariant under reparametrisation. The boundaries of the membrane lie on the five brane where the Green-Schwarz fermions are chiral. These chiral fermions are the source of a reparametrisation anomaly. This anomaly is cancelled by the classical variation provided that membrane tension, the five-brane tension and the eleven-dimensional gravitational constant are suitably related. It is remarkable that these relations are in agreement with other approaches [11, 28, 29, 18]. Together with the Horava-Witten configuration this gives a comprehensive picture of the role of open membranes in relation to M-Theory.

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